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Anomalies, D-flatness and Small Instantons

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ABSTRACT

Recently, Witten has proposed a mechanism for symmetry enhancement in $SO(32)$ heterotic string theory, where the singularity obtained by shrinking an instanton to zero size is resolved by the appearance of an $Sp(1)$ gauge symmetry. In this short letter, we consider spacetime constraints from anomaly cancellation in six dimensions and D-flatness and demonstrate a subtlety which arises in the moduli space when many instantons are shrunk to zero size.

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In a recent paper, Witten[1] has described a new mechanism for enlarging the rank of the gauge group of field theories arising from $SO(32)$ heterotic strings. One considers a six-dimensional compactification on a $K3$ surface, which can be taken to be at a generic point in moduli space. From the index theorem, we know that we must choose a nontrivial gauge bundle V with instanton number $c_2(V) = 24$. There are many ways to do this, and one can describe the situation in terms of an $SO(N)$ gauge configuration, for some $4 \leq N \leq 24$. More precisely, the structure group of an instanton is $SU(2)$ and there are 24 such instantons embedded in the $SO(32)$ group in some way. The details of this embedding determine the unbroken gauge group, or at least the part described by conformal field theory.

In Ref. [1], Witten considered the singularity obtained when the instanton scale size approaches zero. It was argued that this singularity may be resolved in terms of an enhanced $Sp(1)$ gauge symmetry. The full gauge group then consists of this $Sp(1)$ factor times some subgroup of $SO(32)$, determined by the nature of the remaining gauge bundle. This may be done to any number of the instantons and in the special case where all 24 instantons have been shrunk to zero size, the gauge group is $SO(32) \times Sp(1)^{24}$. This may be enhanced to $SO(32) \times Sp(24)$ by bringing the positions of the point instantons together. As Witten showed, this has a satisfying Type I dual description in terms of D-branes.[2] For each point instanton, there is a 5-brane which carries $Sp(1)$ Chan-Paton factors.[1] As k of the 5-branes come together, there are additional light Dirichlet open string states which fill out a vector multiplet of $Sp(k)$.

Having compactified on $K3$, the low-energy field theory possesses a 6-dimensional $N = 1$ sypersymmetry.[2] It is well known that anomaly cancellation in 6 dimensions is quite restrictive. In this letter, we wish to enlarge on the discussion of Ref. [1] paying particular attention to anomaly cancellation and D-flatness. We will reconstruct the spectrum of the theory of maximal symmetry and demonstrate its (Green-Schwarz) anomaly cancellation. Next, we study the moduli space by solving the D-flatness constraints. These constraints turn out to be somewhat non-trivial and indicate which field theories may be accessed

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[1] The world-brane field theory contains $Sp(1)$ super Yang Mills.

[2] Corresponding to $N = 2$ in $D = 4$.

by Higgsing. Of course, Higgsing (along a flat direction!) must not spoil the anomaly cancellation and we demonstrate that this is so. The resulting theories may be given an interpretation in terms of D-branes: these theories may be understood[3] as consistent compactifications of open strings on $K3$ orbifolds.

1. Anomaly Cancellation

The theory of maximal symmetry given by Witten has gauge group $SO(32) \times Sp(24)$. The hypermultiplets arrange themselves in the representations

$$
\begin{aligned}
ND : &\quad \frac{1}{2} \times (\mathbf{32}, \mathbf{48}) \\
DD : &\quad (\mathbf{1}, \boxminus) \\
K3 : &\quad 20 \times (\mathbf{1}, \mathbf{1}).
\end{aligned} \tag{1.1}
$$

The labels such as 'ND' refer to the labeling of states[3] in the D-brane representation. The 20 singlet states may be understood as the 4 hypermultiplets of the internal g_{ij}, B_{ij} plus the 16 blowing-up modes. The total number of hypermultiplets ($n_H = 768 + 1128 + 20 = 1916$) is 244 more than the number of vector multiplets, and thus the $\mathrm{tr}R^4$ anomaly cancels.[4],[5] In addition, the $\mathrm{tr}F^4$ anomaly terms cancel. The remaining mixed gravitational-gauge anomalies will not cancel, but can be eliminated by the Green-Schwarz mechanism provided they factorize in a suitable way. In the present case, we find (see also Ref. [6])

$$
I \propto (\mathrm{tr}R^2 + 2\mathrm{tr}F_{32}^2 - 2\mathrm{tr}F_{24}^2) \times (\mathrm{tr}R^2 - \mathrm{tr}F_{32}^2) \tag{1.2}
$$

and so indeed the model is anomaly-free.

By turning on the antisymmetric DD field, we may break the $Sp(24)$ to a product of symplectic groups with total rank 24. This Higgsing is always D-flat, and we will for simplicity now consider the $SO(32) \times Sp(1)^{24}$ theory with hypermultiplets

$$
\begin{aligned}
&\frac{1}{2} \times (\mathbf{32}, \mathbf{2}, \mathbf{1}^{23}) \quad + \text{permutations} \\
&44 \times (\mathbf{1}, \mathbf{1}^{24})
\end{aligned} \tag{1.3}
$$

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[3] We are counting full hypermultiplets. The factor $1/2$ in the first line of eq. (1.1) refers to the fact that the irreducible supersymmetry representation of a field in a pseudoreal representation of the gauge group contains one independent bosonic field (on-shell).

where the 44 singlets come from the 20 original singlets plus 24 Higgs modes which are uneaten. One may easily check that the anomalies cancel, with the mixed anomalies now factorizing as

$$I \propto \left(\text{tr} R^2 + 2\text{tr} F_{32}^2 - 2 \sum_i \text{tr} F_i^2 \right) \times \left(\text{tr} R^2 - \text{tr} F_{32}^2 \right). \tag{1.4}$$

In the remainder of the paper, we will consider theories obtainable through consistent Higgsing of this model.

2. D-flatness

Several general aspects of the flat directions were discussed in Ref. [1]. The $D = 6$, $N = 1$ supersymmetry algebra contains an $SU(2)_R$ symmetry and the D-terms transform in the triplet representation:

$$D_{ij}^a = \sum_k T_{AB}^a \phi_i^{A\alpha_k} \phi_j^{B\beta_k} \epsilon_{\alpha\beta}$$
$$D_{ij}^{\tilde{a}_k} = T_{\alpha_k\beta_k}^{\tilde{a}_k} \phi_i^{A\alpha_k} \phi_j^{A\beta_k} \tag{2.1}$$

for $k = 1, \ldots, 24$. Here, i, j are $SU(2)_R$ indices, $a; A, B$ are $SO(32)$ indices, $\tilde{a}_k; \alpha_k, \beta_k$ are indices of the kth $Sp(1)$ factor and ϕ are the non-singlet fields of eq. (1.3). In the case at hand ϕ will also satisfy a reality condition

$$\overline{\phi}_{\alpha_k}^{A,i} = \epsilon_{\alpha_k\beta_k} \epsilon^{ij} \phi_j^{A\beta_k}. \tag{2.2}$$

The multiplicity of D-term conditions may be simply understood in terms of the D- and F-flatness conditions of the corresponding dimensionally reduced $D = 4$, $N = 2$ theory.

We consider the case where $SO(28)$ is unbroken; the more general case will be considered shortly. The general picture advocated in Ref. [1] is that the shrinking of instanton scale sizes should be understood in terms of the appearance of extra $Sp(1)$ factors. The converse is also true, but with some important caveats. In particular, if we start with the $SO(32) \times Sp(1)^{24}$ theory given above, we cannot unshrink a *single* instanton: this is not a flat direction of the theory. In fact, the minimal operation is an unshrinking of *four* instantons, as we will now show.

This may be demonstrated by explicitly examining the D-terms when just one of the $Sp(1)$'s is Higgsed. One finds that there are no flat directions satisfying all of the constraints (2.1). Indeed, focus on a single such field ϕ, which is charged, say, under the first $Sp(1)$ factor. Using just the reality condition and the $Sp(1)$ D-flatness condition, one finds nine real conditions on 14 real parameters (recall that we are assuming that $SO(28)$ is unbroken, and we have used our freedom of $Sp(1)$ transformations to eliminate one complex parameter. For later use (where more than one vev will be considered), it will be convenient to not perform $SO(4) \subset SO(32)$ rotations. Thus the solutions to those equations can be parameterized by 5 real parameters (some of which are gauge redundant in the present case). However, we must also satisfy the $SO(4) \subset SO(32)$ D-flatness conditions. It can easily be seen that these constitute $6 \times 3 = 18$ real conditions, and we do not expect a solution; this can be born out by direct computation.

If one turns on n such fields, the $Sp(1)$ D-flat solutions are parameterized by $5n$ real parameters. Restricting to those solutions which are also $SO(4)$ D-flat leaves $5n - 18$ parameters. Generically then, one must turn on at least $n \geq 4$ such fields; the unbroken symmetry in such a vacuum is $SO(28) \times SU(2) \times Sp(1)^{24-n}$. Four fields transforming as the **28** of $SO(28)$ are eaten, as well as some $SO(28)$ singlets. We are left with the following spectrum:

$$
\begin{aligned}
\frac{1}{2} &\times (\mathbf{28}, \mathbf{1}; \mathbf{2}, \mathbf{1}^{23-n}) \quad + \text{permutations} \\
\frac{(n-4)}{2} &\times (\mathbf{28}, \mathbf{2}; \mathbf{1}^{24-n}) \\
&\quad (\mathbf{1}, \mathbf{2}; \mathbf{2}, \mathbf{1}^{23-n}) \quad + \text{permutations} \\
(n-3+44) &\times (\mathbf{1}, \mathbf{1}; \mathbf{1}^{24-n})
\end{aligned}
\tag{2.3}
$$

The total number of hypermultiplets is $n_H = 56(24-n)+28(n-4)+4(24-n)+(n+41) = 697 - 3n$, and there are $n_V = 378 + 3 + 3(24 - n) = 453 - 3n$ vectors. Thus, the $\mathrm{tr}R^4$ anomaly has cancelled as it must. Incidentally, the requirement of $n \geq 4$ can also be seen through the $\mathrm{tr}F_{28}^4$ anomaly: one must have precisely 20 fields transforming as the **28** of $SO(28)$; not enough such fields can be eaten until $n \geq 4$. The mixed anomalies can be

shown to factorize as

$$I \propto \left(\text{tr} R^2 + 2 \text{tr} F_{28}^2 - 2(n-2) \text{tr} F_2^2 - 2 \sum_{i=1}^{24-n} \text{tr} F_i^2 \right) \times \left(\text{tr} R^2 - \text{tr} F_{28}^2 - 2 \text{tr} F_2^2 \right) \quad (2.4)$$

and thus can be cancelled by the Green-Schwarz mechanism. The spectrum for $n > 4$ can also be obtained from the $n = 4$ case by turning on some of the $(\mathbf{1}, \mathbf{2}; \mathbf{2}, \mathbf{1}^{19})$ fields present in that case.

3. $SO(N)$ **bundles**

We may also consider $SO(N)$ bundles for $N > 4$. For given N and n, there are two branches, one where the gauge symmetry is $SO(32 - N) \times Sp(1)^{24-n}$, the other where there is an additional $SU(2)$. We consider the first case. The D-flatness conditions lead us to turn on $n \geq N$ vevs, and the following spectrum may be shown to be anomaly-free:

$$\frac{1}{2} \times (\mathbf{32} - \mathbf{N}; \mathbf{2}, \mathbf{1}^{23-n}) \quad + \text{permutations}$$
$$(n - N) \times (\mathbf{32} - \mathbf{N}; \mathbf{1}^{24-n})$$
$$\frac{N}{2} \times (\mathbf{1}; \mathbf{2}, \mathbf{1}^{23-n}) \quad + \text{permutations} \quad (3.1)$$
$$(44 - n + x_n) \times (\mathbf{1}; \mathbf{1}^{24-n})$$

where $x_n = n(N-2) - \frac{1}{2} N(N-1)$. This number is exactly what is required to interpret this Higgsing in terms of the unshrinking of small instantons: x_n is precisely the dimension of the moduli space of n instantons, that is, the number of deformations of an $SO(N)$ gauge bundle of instanton number n. We count as follows: each instanton comes with a scale size, 4 position coordinates and 3 $SU(2)$ orientations. The possible embeddings of $SU(2)$ in $SO(N)$ are parameterized by the coset $SO(N)/(SO(N-4) \times SO(4))$ of (real) dimension $4(N - 4)$. This corresponds to a total of $(N - 2)$ hypermultiplets for each instanton. Subtracting off the overall $SO(N)$ gauge rotations, we arrive at $n(N - 2) - \frac{1}{2} N(N - 1)$ as the dimension of the n-instanton moduli space. The counting of singlet fields is also nicely interpreted in terms of D-branes: when the vev of a given field is zero, there is a 5-brane, with a single hypermultiplet describing its position on the compact space. All other fields are in non-trivial representations of the gauge group. When the corresponding

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vev is turned on (i.e. an instanton unshrunk), the 5-brane disappears (accounting for the $-n$ in the number of singlets), and x_n moduli describe the resulting finite-size instanton.

There is an additional branch of the moduli space where an enhanced $SU(2)$ gauge symmetry arises. In this case, we find the anomaly-free spectrum in representations of $SO(32 - N) \times SU(2) \times Sp(1)^{24-n}$:

$$
\begin{aligned}
&\frac{1}{2} \times (\mathbf{32 - N}, \mathbf{1}; \mathbf{2}, \mathbf{1}^{23-n}) \quad + \text{permutations} \\
&\frac{(n - N)}{2} \times (\mathbf{32 - N}, \mathbf{2}; \mathbf{1}^{24-n}) \\
&(\mathbf{1}, \mathbf{2}; \mathbf{2}, \mathbf{1}^{23-n}) \quad + \text{permutations} \\
&\frac{(N - 4)}{2} \times (\mathbf{1}, \mathbf{1}; \mathbf{2}, \mathbf{1}^{23-n}) \quad + \text{permutations} \\
&z \times (\mathbf{1}, \mathbf{2}; \mathbf{1}^{24-n}) \quad + \text{permutations} \\
(44 - n + x_n - 2z) &\times (\mathbf{1}, \mathbf{1}; \mathbf{1}^{24-n})
\end{aligned}
\tag{3.2}
$$

where $z = (N - 4)(n - N + 4)/2$. Some of the singlets of eq. (3.1) now transform as doublets of the $SU(2)$.

4. Conclusions

The result of Section 2 is rather surprising given the 5-brane interpretation of Ref. [1]. One may have suspected that the 5-branes can be removed one by one; however, as we have seen, there are important spacetime constraints on this procedure. In any case, we have clarified the picture advocated by Witten, that the scaling of instanton size corresponds to (un)Higgsing. Indeed, this must be so, as this is the only possible non-trivial dynamics at low energies in six dimensions. Further properties of these and related models will be explored in Ref. [3].

References

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